DATASCI 385 Quantitative Finance

Lecture 6: Risk and utility value

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Suggested reading: Investments Ch 6

EMORY

Lecture plan

- Learning from historical data
- Risk aversion and utility values

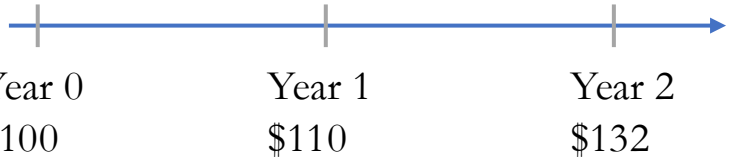
Expected returns

- We can estimate the expected return from historical data
  - Arithmetic average of historic rates of return

$$\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$$

  - Geometric average of historic rates of return

$$Initial\ value \times (1 + g)^n = Terminal\ value \Rightarrow g = \left(\frac{Terminal\ value}{Initial\ value}\right)^{\frac{1}{n}} - 1$$



  - Log return

$$r_i = \log P_i - \log P_{i-1}$$

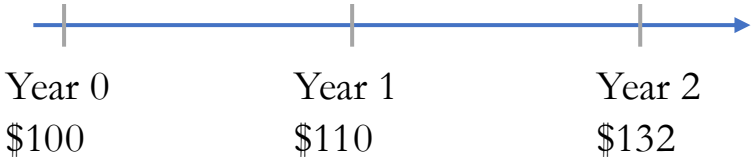
➢ sum of log returns equals to total return: $\sum_{i=1}^{n} r_i = \log P_n - \log P_0$

Estimating variance and standard deviation

- We can estimate the variance of the actual returns from historical data

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2$$

  - $\bar{r} = \frac{1}{n}\sum_{i=1}^{n} r_i$
  - The denominator $n-1$ is to account for the degree of freedom
  - Example
    - $\hat{\sigma}^2 = \frac{1}{2-1}[(0.2-0.15)^2 + (0.1-0.15)^2] = 0.005$
    - Standard deviation $\hat{\sigma} = 0.005^{1/2} = 0.071$



Measuring asymmetry

- Skew measures asymmetry in the probability distribution of returns

$$\text{Skew} = \text{Average}\left[\frac{(r-\bar{r})^3}{\hat{\sigma}^3}\right]$$

  - E.g., large negative returns are more likely than large positive returns
  - Negative skew: extreme *bad* outcomes are more frequent than extreme positive ones (skew to the left, fatter left tail, underestimate risk)
  - Positive skew: opposite case (skew to the right)
  - Skew of Normal distribution is 0

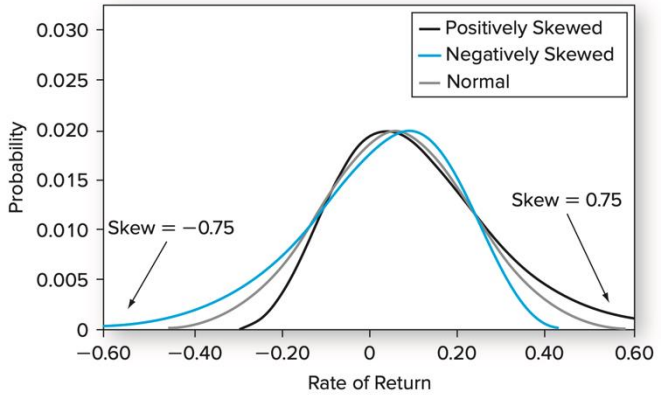


Figure 5.4 Normal and skewed distributions (mean = 6%, SD = 17%)

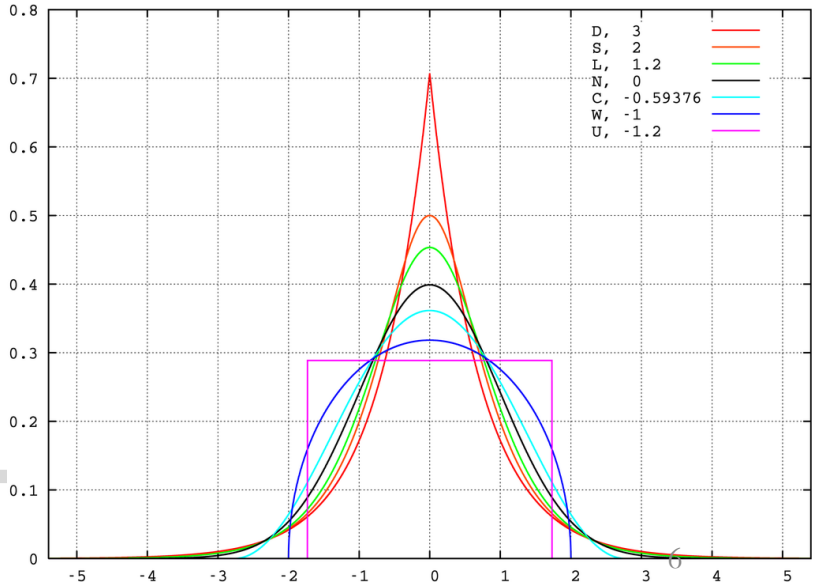
Measuring extreme values

- Kurtosis measures likelihood of extreme values on either side of the mean

$$\text{Kurtosis} = \text{Average}\left[\frac{(r-\bar{r})^4}{\hat{\sigma}^4}\right] - 3$$

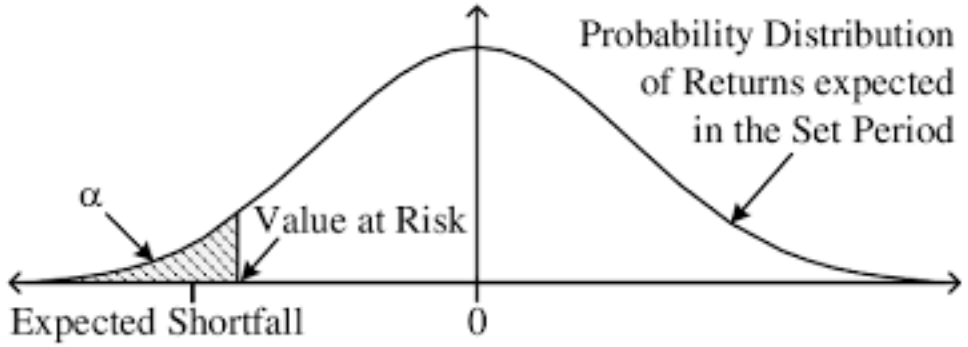
  - Deviations are raised to the fourth power so more sensitive to extreme outcomes
  - Subtract by 3 because Average $\left[\frac{(r-\bar{r})^4}{\hat{\sigma}^4}\right]$ for Normal distribution is 3

Letter	Distribution	Kurtosis
D	Double exponential (Laplace)	3
S	Hyperbolic secant	2
L	Logistic	1.2
N	Normal	0
C	Raised cosine	−0.59376
W	Wigner semicircle	−1
U	Uniform	−1.2



Value at risk

- Practitioners are concerned about vulnerability to large losses
- One measure of downside risk: $q\%$ **value at risk** ($q\%$ VaR)
  - How much would I lose if my return was in the $q\%$ of the distribution
  - **VaR** differs from *Var*, the abbreviation for variance
  - Quantile of a distribution
  - Example: 1% VaR: 99% of returns will exceed the VaR and 1% of returns will be worse



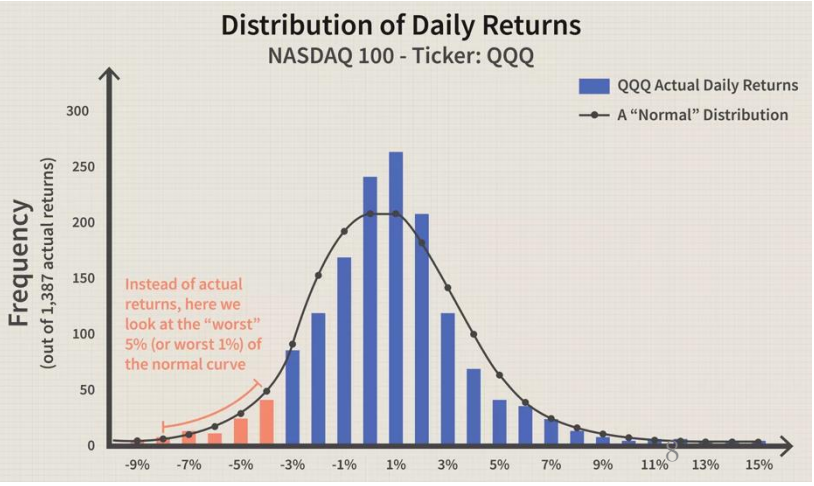
Estimating VaR

- If portfolio/asset returns are normally distribution, then

$$VaR(1\%, normal) = Mean - 2.33\ SD$$

  - -2.33 is the first percentile of the standard normal distribution (with mean = 0 and SD = 1)

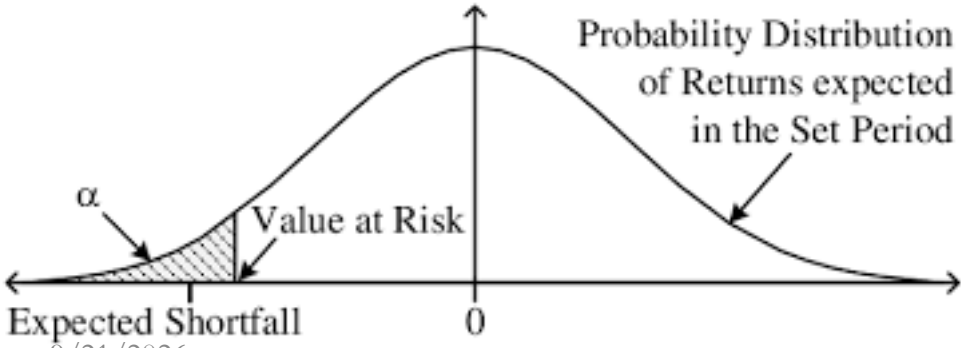
- Estimating VaR from the historical data
  - Sort the observations from high to low
  - VaR is the return at the $q\%$ percentile of the sample distribution

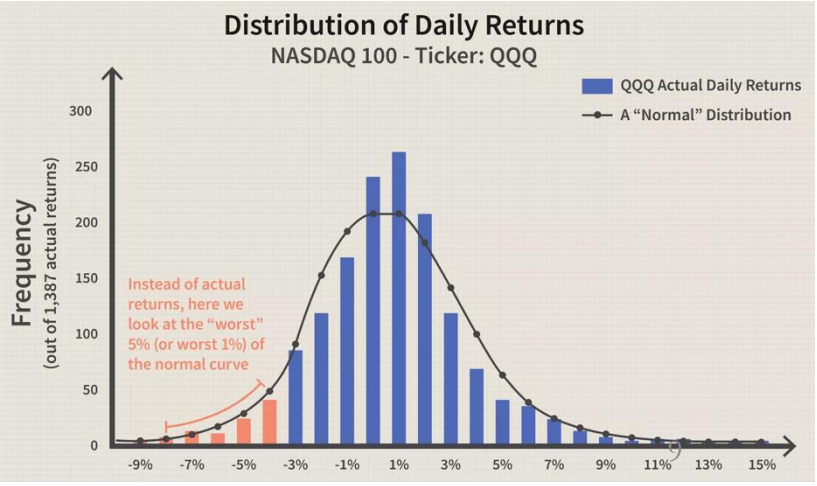


Expected shortfall

- Another measure of downside risk: q% **Expected shortfall (ES)** or **conditional value at risk (CVaR)**
  - Expected loss given that we find ourselves in one of the q% worst-case scenarios
  - As a comparison, q% VaR is the most optimistic measure of bad-case outcomes as it takes highest return (smallest loss) of all these cases

- Estimating ES from the historical data
  - 1% ES: identifying the worst 1% of all observations and taking their average





Exercise

- The table lists 100 daily stock returns (%), sorted high to low down columns A–D. Compute the 5% VaR and 5% expected shortfall.

	A	B	C	D
1	Stock Return	Stock Return	Stock Return	Stock Return (%)
2	3.8	0.85	-0.18	-1.1
3	3.26	0.82	-0.27	-1.19
4	3.23	0.82	-0.34	-1.25
5	3.18	0.81	-0.35	-1.3
6	3.15	0.79	-0.37	-1.34
7	3.06	0.76	-0.37	-1.52
8	3.03	0.76	-0.37	-1.58
9	2.81	0.75	-0.48	-1.72
10	2.21	0.73	-0.5	-1.93
11	2.16	0.69	-0.5	-2.02
12	2.11	0.62	-0.52	-2.11
13	2.05	0.58	-0.56	-2.2
14	2.04	0.52	-0.67	-2.29
15	1.96	0.49	-0.68	-2.34
16	1.93	0.44	-0.82	-2.56
17	1.75	0.32	-0.83	-2.72
18	1.74	0.29	-0.83	-2.75
19	1.73	0.28	-0.84	-2.83
20	1.63	0.27	-0.86	-2.86
21	1.58	0.24	-0.9	-3.35
22	1.4	0.11	-0.94	-3.43
23	1.32	0.07	-0.96	-3.73
24	1.19	0.03	-0.99	-3.82
25	1.13	-0.04	-1.02	-3.88
26	1.09	-0.13	-1.1	-5.14

Lecture plan

- Learning from historical data

- Risk aversion and utility values

Thought exercise

- What is the Sharpe ratio of each portfolio?

- Which portfolio would you choose?

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

Table 6.1

Available risky portfolios (risk-free rate = 5%)

Risk aversion and utility values

- **Risk-averse** investor penalizes the expected rate of return by the risk involved
  - A portfolio is more attractive when its expected return is higher and its risk is lower
- **Utility score** to compare competing portfolios based on expected return and risk of those portfolios

$$U = E(r) - \frac{1}{2}A\sigma^2$$

  - U: utility value
  - A: index of the investor's risk aversion
    - Risk-averse investors: $A > 0$
    - Risk-neural investors: $A = 0$
    - Risk-lover investors: $A < 0$
  - Quantify the rate at which investors are willing to trade off return against risk

Estimating *A*

- The questionnaire is an example

WORDS FROM THE STREET

What Level of Risk Is Right for You?

No risk, no reward. Most people intuitively understand that they have to bear some risk to achieve an acceptable return on their investment portfolios.

But how much risk is right for you? If your investments turn sour, you may put at jeopardy your ability to retire, to pay for your kid's college education, or to weather an unexpected need for cash. These worst-case scenarios focus our attention on how to manage our exposure to uncertainty.

Assessing—and quantifying—risk aversion is, to put it mildly, difficult. It requires confronting at least these two big questions.

First, how much investment risk can you afford to take? If you have a steady high-paying job, for example, you have greater ability to withstand investment losses. Conversely, if you are close to retirement, you have less ability to adjust your lifestyle in response to bad investment outcomes.

Second, you need to think about your personality and decide how much risk you can tolerate. At what point will you be unable to sleep at night?

To help clients quantify their risk aversion, many financial firms have designed quizzes to help people determine whether they are conservative, moderate, or aggressive investors. These quizzes try to get at clients' attitudes toward risk and their capacity to absorb investment losses.

Here is a sample of the sort of questions that can shed light on an investor's risk tolerance.

MEASURING YOUR RISK TOLERANCE

Circle the letter that corresponds to your answer.

1. The stock market fell by more than 30% in 2008. If you had been holding a substantial stock investment in that year, which of the following would you have done?
   a. Sold off the remainder of your investment before it had the chance to fall further.
   b. Stayed the course with neither redemptions nor purchases.
   c. Bought more stock, reasoning that the market is now cheaper and therefore offers better deals.
2. The value of one of the funds in your 401(k) plan (your primary source of retirement savings) increased 30% last year. What will you do?
   a. Move your funds into a money market account in case the price gains reverse.
   b. Sit tight and do nothing.
   c. Put more of your assets into that fund, reasoning that its value is clearly trending upward.
3. How would you describe your non-investment sources of income (for example, your salary)?
   a. Highly uncertain
   b. Moderately stable
   c. Highly stable
4. At the end of the month, you find yourself:
   a. Short of cash and impatiently waiting for your next paycheck.
   b. Not overspending your salary, but not saving very much.
   c. With a comfortable surplus of funds to put into your savings account.
5. You are 30 years old and enrolling in your company's retirement plan, and you need to allocate your contributions across 3 funds: a money market account, a bond fund, and a stock fund. Which of these allocations sounds best to you?
   a. Invest everything in a safe money-market fund.
   b. Split your money evenly between the bond fund and stock fund.
   c. Put everything into the stock fund, reasoning that by the time you retire, the year-to-year fluctuations in stock, returns will have evened out.
6. You are a contestant on *Let's Make a Deal,* and have just won $1,000. But you can exchange the winnings for two random payoffs. One is a coin flip with a payoff of $2,500 if the coin comes up heads. The other is a flip of two coins with a payoff of $6,000 if both coins come up heads. What will you do?
   a. Keep the $1,000 in cash.
   b. Choose the single coin toss.
   c. Choose the double coin toss.
7. Suppose you have the opportunity to invest in a start-up firm. If the firm is successful, you will multiply your investment by a factor of ten. But if it fails, you will lose everything. You think the odds of success are around 20%. How much would you be willing to invest in the start-up?
   a. Nothing
   b. 2 months' salary
   c. 6 months' salary
8. Now imagine that to buy into the start-up you will need to borrow money. Would you be willing to take out a $10,000 loan to make the investment?
   a. No
   b. Maybe
   c. Yes

SCORING YOUR RISK TOLERANCE

For each question, give yourself one point if you answered (*a*), two points if you answered (*b*), and three points for a (c). The higher your total score, the greater is your risk tolerance, or, equivalently, the lower is your risk aversion.

Evaluating investments by using utility scores

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

Table 6.1

Available risky portfolios (risk-free rate = 5%)

Investor Risk Aversion (*A*)	Utility Score of Portfolio *L* [$E(r) = 0.07; \sigma = 0.05$]	Utility Score of Portfolio *M* [$E(r) = 0.09; \sigma = 0.10$]	Utility Score of Portfolio *H* [$E(r) = 0.13; \sigma = 0.20$]
2.0	$0.07 - \frac{1}{2} \times 2 \times 0.05^2 = 0.0675$	$0.09 - \frac{1}{2} \times 2 \times 0.1^2 = 0.0800$	**$0.13 - \frac{1}{2} \times 2 \times 0.2^2 = 0.09$**
3.5	$0.07 - \frac{1}{2} \times 3.5 \times 0.05^2 = 0.0656$	**$0.09 - \frac{1}{2} \times 3.5 \times 0.1^2 = 0.0725$**	$0.13 - \frac{1}{2} \times 3.5 \times 0.2^2 = 0.06$
5.0	$0.07 - \frac{1}{2} \times 5 \times 0.05^2 = 0.0638$	**$0.09 - \frac{1}{2} \times 5 \times 0.1^2 = 0.0650$**	$0.13 - \frac{1}{2} \times 5 \times 0.2^2 = 0.03$

Interpretation of utility score

- The utility score of risky portfolios can be interpreted as a **certainty equivalent** rate of return

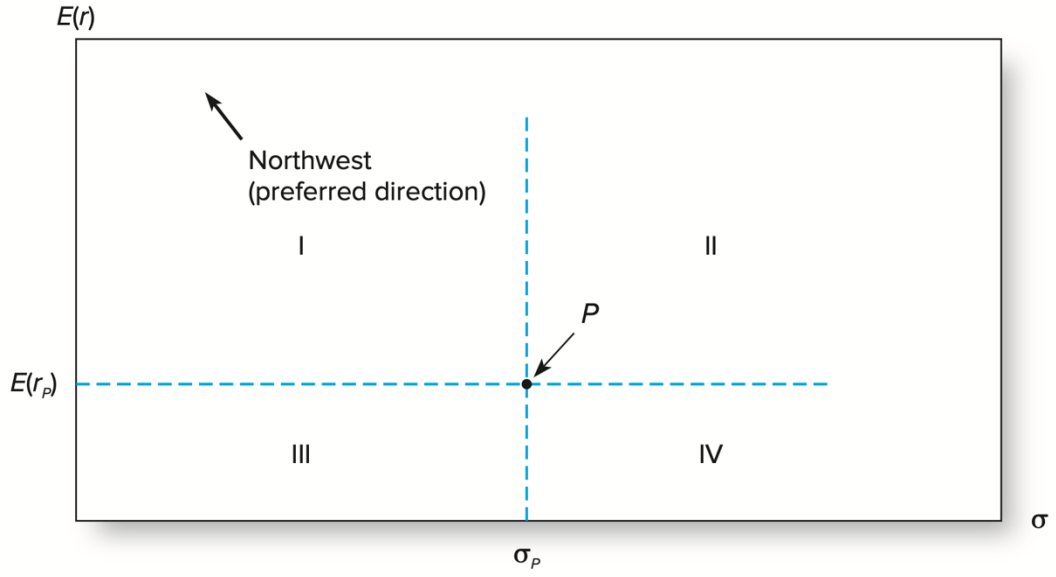
- Certainty equivalent is the rate that risk-free investment would need to offer to provide the same utility as the risky portfolio
  - A natural way to rank competing portfolios
  - A portfolio is desirable only if its certainty equivalent exceeds that of the risk-free equivalent

Question

- Question: A portfolio has an expected rate of return of 20% and standard deviation of 30%. T-bills offer a safe rate of return of 7%. Would an investor with risk-aversion parameter $A = 4$ prefer to invest in T-bills or the risky portfolio? What if $A = 2$?

Risk and return tradeoff

- Portfolio *P* dominates any portfolio in quadrant IV
- Any portfolio in quadrant I dominates portfolio *P*
- Portfolios in quadrant II and III depend on investor's risk aversion



Indifference curve

- Indifference curve connects all portfolio points with the same utility value

