DATASCI 385 Quantitative Finance

Lecture 12: APT and empirical analysis

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Suggested reading: Investments Ch 10

EMORY

Arbitrage pricing theory

- Stephen Ross developed the arbitrage pricing theory (APT) in 1976

- Like the CAPM, the APT predicts a security market line linking expected returns to risk

- Three key propositions
  1. Security returns can be described by a factor model
  2. There are sufficient securities to diversify away idiosyncratic risk
  3. Well-functioning security markets do not allow arbitrage opportunities to persist

Multifactor arbitrage pricing theory

- Consider a two-factor model for portfolio P

$$R_P = E(R_P) + \beta_P F = E(R_P) + \beta_{P1} F_1 + \beta_{P2} F_2$$

  - $F = [F_1, F_2]'$ and $\beta_P = [\beta_{P1}, \beta_{P2}]$
  - For example
    - F_1: ΔGDP, the departure of GDP growth from expectations
    - F_2: ΔIR, the unanticipated change in interest rates
  - Factor portfolios: benchmark (well-diversified) portfolios (or tracking portfolios) in the APT, have a beta of one on one of the factors and a beta of zero on any other factor

Risk premium for multifactor APT

- $E(R_1) = E(r_1) - r_f$: risk premium of the first factor portfolio
- $E(R_2) = E(r_2) - r_f$: risk premium of the second factor portfolio

- The risk premium of any portfolio P

$$E(R_P) = \beta_P E(F) = \beta_{P1} \cdot E(R_1) + \beta_{P2} \cdot E(R_2)$$
$$\Rightarrow E(r_P) = r_f + \beta_{P1} \cdot \left(E(r_1) - r_f\right) + \beta_{P2} \cdot \left(E(r_2) - r_f\right)$$

Fama-French (FF) Three-Factor Model

- Identify the most likely sources of systematic risk: Use firm characteristics for security returns

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it}$$

  - SMB = Small Minus Big (i.e., the return of a portfolio of small stocks in excess of the return on a portfolio of large stocks)

  - HML = High Minus Low (i.e., the return of a portfolio of stocks with a high book-to-market ratio in excess of the return on a portfolio of stocks with a low book-to-market ratio)

$$\text{Book} - \text{to} - \text{market ratio} = \frac{\text{Book value per share}}{\text{Market value per share}}$$

    - Book value is calculated from the balance sheet as total assets minus total liabilities

Fama-French (FF) Three-Factor Model

$$R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + e_{it}$$

- These variables may proxy for hard-to-measure more-fundamental variables
- HML: Firms with high book-to-market ratios are more likely to be in financial distress
- SMB: Small stocks may be more sensitive to changes in business conditions

Portfolio sorting

- Use firm size as an example
  - Step 1: Use June market capitalization each year to measure size
  - Step 2: Assign NYSE stocks to portfolio small, 2, 3, 4, big
  - Step 3: Hold the portfolios from July in Year s to June in Year $s+1$ (either value-weighted or equally-weighted)
  - Step 4: Construct return: $SMB_t = R_{small,t} - R_{big,t}$

  - Rebalance every year

Small-firm effect

- Small-firm effect was originally documented by Banz (1981)
- Two explanations
  - Neglected-firm effect: Small firms tend to be neglected by large institutional traders (Arbel and Strebel (1983))
  - Liquidity effect: Small firms have higher bid-ask spread and entail higher trading costs (Amihud and Mendelson (1986))

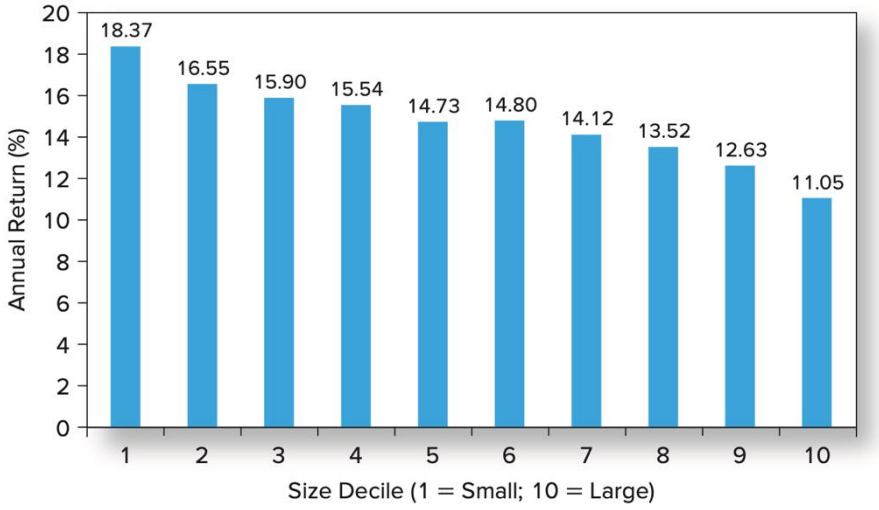


Figure 11.3 Average annual return for 10 size-based portfolios, 1926–2018

Source: Authors' calculations, using data obtained from Professor Ken French's data library at **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

Journal of Financial Economics 9 (1981) 3–18. North-Holland Publishing Company

THE RELATIONSHIP BETWEEN RETURN AND MARKET VALUE OF COMMON STOCKS*

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Received June 1979, final version received September 1980

This study examines the empirical relationship between the return and the total market value of NYSE common stocks. It is found that smaller firms have had higher risk adjusted returns, on average, than larger firms. This 'size effect' has been in existence for at least forty years and is evidence that the capital asset pricing model is misspecified. The size effect is not linear in the market value; the main effect occurs for very small firms while there is little difference in return between average sized and large firms. It is not known whether size *per se* is responsible for the effect or whether size is just a proxy for one or more true unknown factors correlated with size.

Book-to-market ratios

- Portfolios of higher book-to-market ratio have higher returns than portfolios of low book-to-market ratio (Fama and French (1992))

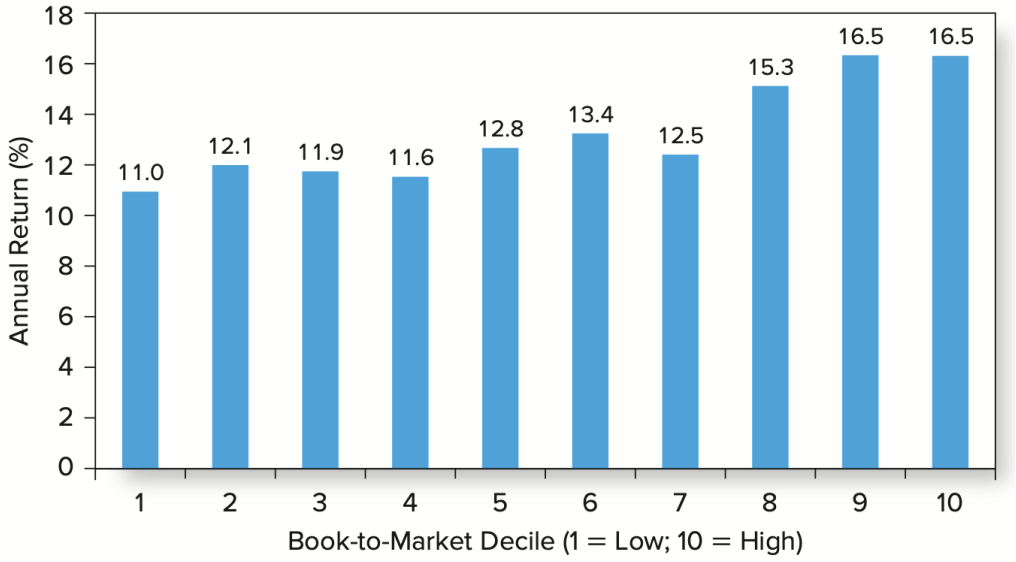


Figure 11.4 Average return as a function of book-to-market ratio, 1926–2018

Source: Authors' calculations, using data obtained from Professor Ken French's data library at **http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.**

Journal of Financial Economics 33 (1993) 3–56. North-Holland

Common risk factors in the returns on stocks and bonds*

Eugene F. Fama and Kenneth R. French
University of Chicago, Chicago, IL 60637, USA

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This paper identifies five common risk factors in the returns on stocks and bonds. There are three stock-market factors: an overall market factor and factors related to firm size and book-to-market equity. There are two bond-market factors, related to maturity and default risks. Stock returns have shared variation due to the stock-market factors, and they are linked to bond returns through shared variation in the bond-market factors. Except for low-grade corporates, the bond-market factors capture the common variation in bond returns. Most important, the five factors seem to explain average returns on stocks and bonds.

Estimating and implementing a three-factor SML

- The equilibrium expected rate of return on Amazon stock

$$r_{amazon,t} - r_{ft} = \alpha_{amazon} + \beta_M(r_{M,t} - r_{ft}) + \beta_{HML}HML_t + \beta_{SMB}SMB_t + e_{amazon,t}$$

  - Risk-free rate $r_{ft} = 1\%$
  - Market risk premium $E(r_M) - r_f = 6\%$
  - Risk premium on SMB $E(R_{SMB}) = E(r_{SMB}) - r_f = 2\%$
  - Risk premium on HML $E(R_{HML}) = E(r_{HML}) - r_f = 2\%$

Estimating and implementing a three-factor SML

- Fitted three-factor model

$$E(r_{amazon}) = r_f + \beta_M\left(E(r_M) - r_f\right) + \beta_{SMB}\left(E(r_{SMB}) - r_f\right) + \beta_{HML}\left(E(r_{HML}) - r_f\right)$$
$$= 1\% + (1.612\times 6\%) + (-.689\times 2\%) + (-1.133\times 2\%) = 7.028\%$$

  - the considerable hedging value it offers against the size and value risk factors (risk premium lower than the single factor model)

- Fitted single-factor model

$$E(r_{amazon}) = r_f + \beta_M\left(E(r_M) - r_f\right)$$
$$= 1\% + (1.533\times 6\%) = 10.198\%$$

	Single-Factor Model		Three-Factor Model	
	Regression Coefficient	*t*-Statistic	Regression Coefficient	*t*-Statistic
Intercept (alpha)	1.916%	2.065	1.494%	1.790
$r_M - r_f$	1.533	4.865	1.612	5.866
SMB			−0.689	−2.126
HML			−1.133	−3.304
R-square	.286		.455	
Residual std. dev.	6.864%		6.101%	

Other factors and smart-beta ETF

- Other factors and smart-beta ETFs: exposure to specific characteristics such as value, growth, or volatility. E.g.,
  - Size (SMB)
  - Value (HML)
  - Momentum (WML, for Winners Minus Losers): the return on a portfolio that buys recent well-performing stocks and sells poorly performing ones
  - Volatility: the standard deviation of stock returns
  - Quality (profitability): the difference in returns of stocks with high versus low return on assets or similar measures of profitability
  - Investment: the difference between returns on firms with high versus low rates of asset growth
  - Dividend yield

Characteristic-based Fama-MacBeth regression

- Regress firm's return on firm characteristics for each month
- $R_{i,t+1} = \alpha_t + \beta_{1,t}\, Size_{i,t} + \beta_{2,t}\, BM_{i,t} + \beta_{3,t}\, Momentum_{i,t} + \cdots + \epsilon_{i,t}$
- The following table reports $\bar{\beta}_1 = \frac{1}{T}\sum_{t=1}^{T} \beta_{1,t}$, $\bar{\beta}_2 = \frac{1}{T}\sum_{t=1}^{T} \beta_{2,t}$, $\cdots$

Table 13.4

The Cross Section of Expected Stock Returns

	Slope	*t*-Statistic
Size: Stock market capitalization	–.15	–5.01
Book-to-market ratio	.35	6.18
Momentum: Return in past year	.96	6.86
Stock issues: Growth in shares outstanding	–.35	–3.52
Accruals: Change in net working capital	–1.38	–5.69
Profitability: Return on assets	1.43	3.57
Asset growth: Growth in total assets	–.54	–4.49
Dividend yield	–.46	–.27
Beta: Single index model	.33	3.05
Volatility: Standard deviation of stock returns	–1.45	–1.40
Turnover: Shares traded as a fraction of outstanding shares	–4.49	–3.68
Sales: Total sales as multiple of market capitalization	.04	3.10

Source: Jonathan Lewellen, "The Cross-Section of Expected Stock Returns," *Critical Finance Review,* 4 (2015), pp. 1–44.

Latent factor model

- Consider a latent multifactor model

$$R_P = E(R_P) + \beta_{P1}F_1 + \beta_{P2}F_2 + \cdots + \beta_{Pk}F_k$$

  - $F_1, F_2, \cdots, F_k$ are unobserved
  - Estimate $F_1, F_2, \cdots, F_k$ from the data (e.g., using principal component analysis)
  - F_1 is strongly correlated with the market factor

- Risk premium of portfolio P is decomposed as

$$E(R_P) = \beta_{P1}E[R_1] + \beta_{P2}E[R_2] + \cdots + \beta_{Pk}E[R_k]$$

  - $E[R_1]$ is the risk premium of F_1, …, $E[R_k]$ is the risk premium of F_k

Fama-MacBeth regression

- Fama-MacBeth regression estimate betas and risk premia for risk factors
- Step 1: Regress each of n asset returns against m risk factors to determine each asset's beta exposure

$$R_{1,t} = \alpha_1 + \beta_{1,F_1} F_{1,t} + \beta_{1,F_2} F_{2,t} + \cdots + \beta_{1,F_m} F_{m,t} + \epsilon_{1,t}$$

$$R_{2,t} = \alpha_2 + \beta_{2,F_1} F_{1,t} + \beta_{2,F_2} F_{2,t} + \cdots + \beta_{2,F_m} F_{m,t} + \epsilon_{2,t}$$

$$\dots$$

$$R_{n,t} = \alpha_n + \beta_{n,F_1} F_{1,t} + \beta_{n,F_2} F_{2,t} + \cdots + \beta_{n,F_m} F_{m,t} + \epsilon_{n,t}$$

Fama-MacBeth regression

- Step 2: Regress all asset returns for each of T time periods against the previously estimated betas to determine the factor return

$$R_{i,1} = \gamma_{1,0} + \gamma_{1,1}\hat{\beta}_{i,F_1} + \gamma_{1,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{1,m}\hat{\beta}_{i,F_m} + \epsilon_{i,1}$$

$$R_{i,2} = \gamma_{2,0} + \gamma_{2,1}\hat{\beta}_{i,F_1} + \gamma_{2,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{2,m}\hat{\beta}_{i,F_m} + \epsilon_{i,2}$$

$$\ldots$$

$$R_{i,T} = \gamma_{T,0} + \gamma_{T,1}\hat{\beta}_{i,F_1} + \gamma_{T,2}\hat{\beta}_{i,F_2} + \cdots + \gamma_{T,m}\hat{\beta}_{i,F_m} + \epsilon_{i,T}$$

- Risk premium of each factor $\bar{\gamma}_1 = \frac{1}{T}\sum_{t=1}^{T}\gamma_{t,1}, \cdots, \bar{\gamma}_m = \frac{1}{T}\sum_{t=1}^{T}\gamma_{t,m}$