DATASCI 385 Quantitative Finance

Lecture 4: Derivatives and return

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Suggested reading: Investments Ch 2 & 5

Derivatives

- Derivative contracts, e.g., futures and options, provide payoffs that depend on the values of other variables such as commodity prices, bond and stock prices, interest rates, or market index values

- Their values *derive from* the values of other assets

- Also called contingent claims because their payoffs are contingent on the value of other values

Options

- A call option gives its holder the right to *purchase* an asset for a specified price, called the exercise or strike price, on or before a specified expiration date
  - The holder of the call only exercises the option when the market value of the asset exceeds the exercise price

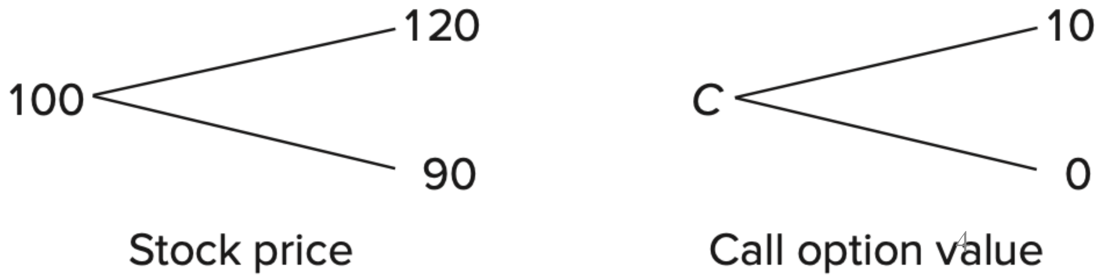
  - American-style option can be exercised at any time prior to or on the expiration date, e.g., all U.S. single-stock options

  - European-style option can only be exercised on the expiration date, e.g., S&P 500 Index options

Call option

- A call option gives its holder the right to *purchase* an asset for a specified exercise price on or before a specified expiration date
  - The holder of the call only exercises the option when the market value of the asset exceeds the exercise price

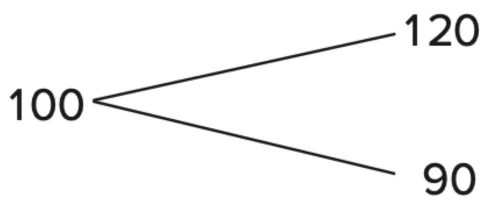
- Example: a February expiration call option on Intel Corp stock with an exercise (or strike) price of $110
  - The current stock price is $100
  - If the stock price rises to $120, the holder exercises and earns $10
  - If the stock price falls to $90, the call is left unexercised



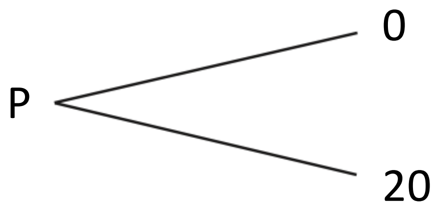
Put option

- A put option gives its holder the right to *sell* an asset for a specified exercise price on or before a specified expiration date
  - The holder of the put only exercises the option when the market value of the asset falls below the exercise price

- Example: a February expiration put option on Intel Corp stock with an exercise (or strike) price of $110
  - The current stock price is $100
  - If the stock price rises to $120, the put is left unexercised
  - If the stock price falls to $90, the holder exercises and earns $20



Stock price



Put option value

Question

- What would be the profit or loss to an investor who bought the January 2019 expiration Shell PLC call option with exercise price $105 if the stock price at the expiration date is $109?

- What about a purchaser of the put option with the same exercise price and expiration?

Valuation of option

- The payoff of the options is always non-negative, so there is a cost or *premium* to own the options

- For both call and put options, each option contract is for the purchase of 100 shares. However, quotations are made on a per-share basis

- Option value
  - Call: decreases (nonlinearly) with strike price, increases with time to expiration
  - Put: increases (nonlinearly) with strike price, increases with time to expiration

Table 2.6

Prices of stock options on Microsoft, January 2, 2019

Expiration	Strike	Call	Put
18-Jan-2019	95	7.65	0.98
18-Jan-2019	100	3.81	2.20
18-Jan-2019	105	1.45	4.79
8-Feb-2019	95	9.50	2.86
8-Feb-2019	100	5.60	3.92
8-Feb-2019	105	3.08	6.35

Note: Microsoft stock on this day was $101.51.
Source: Compiled from data downloaded from Yahoo! Finance.

Futures

- A futures contract calls for delivery of an asset at a specified maturity date for an agreed-upon price, called the futures price, to be paid at contract maturity

- Two parties:
  - *Long position* commits to purchasing the asset on the delivery date
  - *Short position* commits to delivering the asset at contract maturity

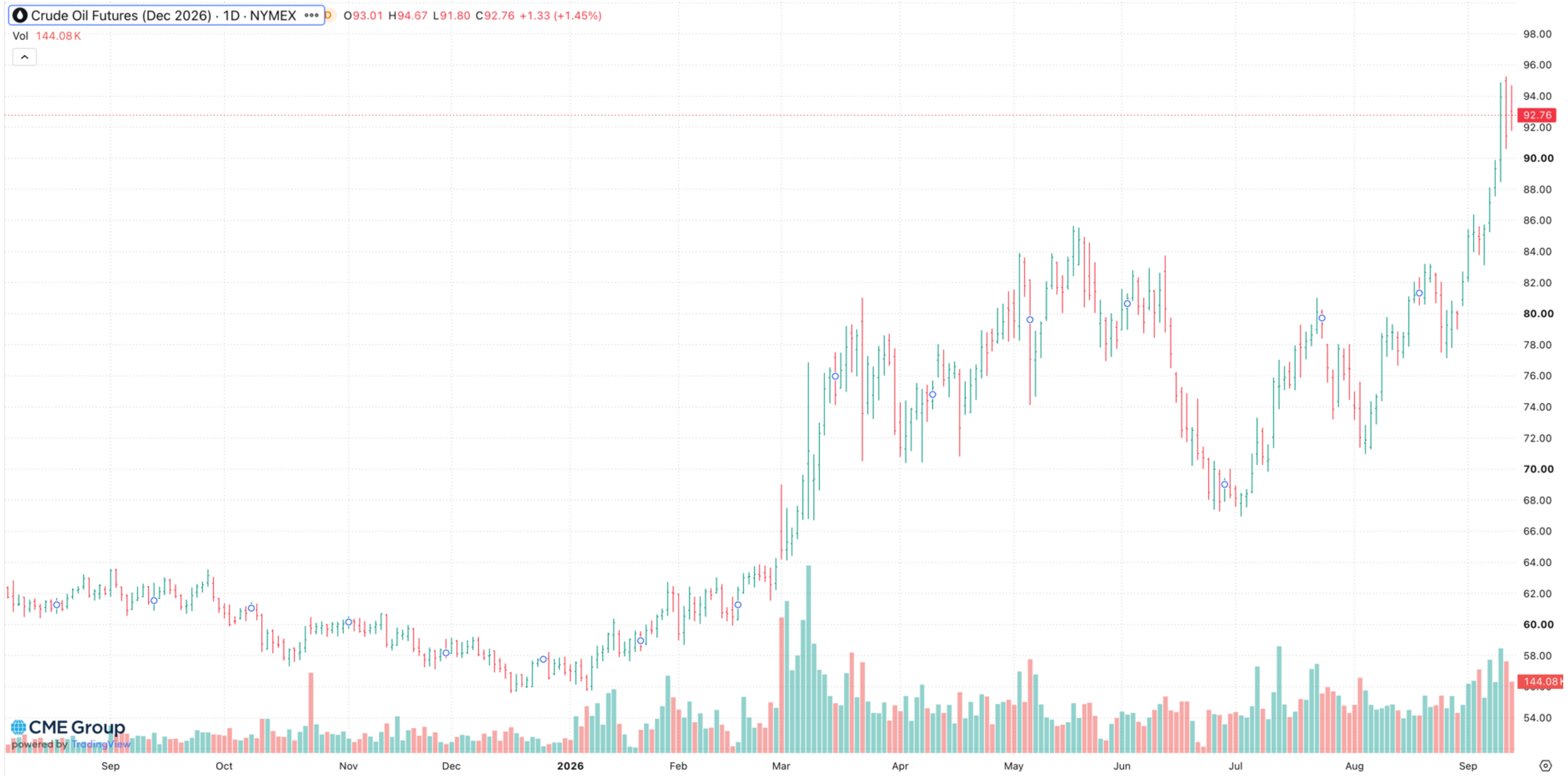
Example

- A future contract calls for delivery of 1,000 barrels of crude oil in December 2026 at the price \$90 per barrel
- Suppose at contract maturity, crude oil is selling for \$100 per barrel
- The profit to the long position: $1{,}000 \times (\$100 - \$90) = \$10{,}000$
- What is the loss to the short position?

Example

Chicago Mercantile Exchange Group (CME) creates benchmark crude oil contracts

Crude oil is trading at $101.60 per barrel (9/14/2026)



US and Israel at war with Iran since 2/28/2026

Options vs futures

- The long position of a futures contract: *obliges* to purchase the asset at the futures price
- The long position (owner) of a call option: *conveys the right* to purchase the asset at the exercise price
- With the same futures price and option's exercise price, the call holder has a better position
  - Call options must be purchased
  - Futures contracts can be entered into without cost

Rate of return

- Rate of return: the percentage change in the value of an investment
- Consider a zero-coupon bond with maturity date T and par value $100
  - Pays its owner only one cash flow, $100, on the maturity date
  - The investor buys the bond with a discount price $P(T)$ (less than $100)
  - Rate of return over the holding period T is

$$r(T) = \frac{100}{P(T)} - 1$$

- Observation: Rate of return generally increases with T

Horizon, T	Price, $P(T)$	$r(T) = \frac{100}{P(T)} - 1$
Half-year	$97.36	100/97.36 – 1 = 0.0271 = 2.71%
1 year	$95.52	100/95.52 – 1 = 0.0469 = 4.69%
25 years	$23.30	100/23.30 – 1 = 3.2918 = 329.18%

Rate of return over a common period

- Effective annual rate (EAR): the percentage increase in funds *per year*
  - Compare returns on investments with differing horizons
  - Accounts for compound return/interest: the cumulative effect that a series of gains or losses has on an amount of capital over time

$$1 + EAR = \left(1 + r(T)\right)^{1/T}$$

Year 0 — $97.36
6 mons — $100
Year 1 — $102.71

$$EAR = \frac{102.71}{97.36} - 1 = 0.0549$$

Horizon, T	Price, $P(T)$	$r(T) = \frac{100}{P(T)} - 1$	EAR over Given Horizon
Half-year	\$97.36	100/97.36 – 1 = 0.0271 = 2.71%	$(1 + .0271)^2 - 1 = .0549$
1 year	\$95.52	100/95.52 – 1 = 0.0469 = 4.69%	$(1 + .0469) - 1 = 0.0469$
25 years	\$23.30	100/23.30 – 1 = 3.2918 = 329.18%	$(1 + 3.2918)^{1/25} - 1 = .060$

$$1 + EAR = \left(1 + r(0.5)\right)^{1/0.5}$$

$$1 + EAR = \left(1 + r(25)\right)^{1/25}$$

Annual percentage rates

- For short-term investments (with holding periods less than a year), rates of return are often annualized using simple interest that ignores compounding, called annual percentage rates (APR)
  - E.g., hold period is 1 month with interest 1.5%, $APR = 12 \times 1.5\% = 18\%$

- With n compounding periods per year, we can find EAR from APR by

$$1 + EAR = \left(1 + \frac{APR}{n}\right)^n$$

- Example: If $APR = 18\%$ and holding period is a month, then

$$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$

EAR vs APR

- The difference between APR and EAR grows with the frequency of compounding

- Suppose $APR = 18\%$.
  - If a compounding period is a month, then
  $$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$
  - If a compounding period is a week, then
  $$EAR = \left(1 + \frac{18\%}{52}\right)^{52} - 1 = 19.68\%$$
  - If a compounding period is a day, then
  $$EAR = \left(1 + \frac{18\%}{365}\right)^{365} - 1 = 19.72\%$$

EAR vs APR

- The difference between APR and EAR grows with the interest rate per period (e.g., month)

- Suppose a compounding period is a month.
  - If $APR = 6\%$, then

$$EAR = \left(1 + \frac{6\%}{12}\right)^{12} - 1 = 6.17\%$$

  - If $APR = 18\%$, then

$$EAR = \left(1 + \frac{18\%}{12}\right)^{12} - 1 = 19.56\%$$

  - If $APR = 48\%$, then

$$EAR = \left(1 + \frac{48\%}{12}\right)^{12} - 1 = 60.10\%$$

Continuous compounding

- As the number of compounding periods n gets larger, we effectively approach *continuous compounding (CC)*
- The continuous compounding rate r_{cc} is defined as the rate that satisfies

$$e^{r_{cc}} = \lim_{n\to\infty}\left(1+\frac{APR}{n}\right)^{n}$$

where $e = 2.71828$ is the Euler's number

- Using the property $\lim_{x\to\infty}(1+x)^{1/x} = e$, we can solve r_{cc}

$$r_{cc} = \lim_{n\to\infty}\log\left(1+\frac{APR}{n}\right)^{n} = \lim_{n\to\infty}\log\left(\left(1+\frac{APR}{n}\right)^{n/APR}\right)^{APR} = \lim_{n\to\infty}\log e^{APR} = APR$$

Total return given continuous compounding

- Given a continuously compounded rate r_{cc}, the total return for any period T is

$$\exp(T \times r_{cc})$$

  - Simplify the calculation

  - For example, $APR = 18\%$ and the investment period is 2 years. Then

$$\lim_{n\to\infty}\left(1+\frac{APR}{n}\right)^{2n} = (\exp(r_{cc}))^2 = \exp(2r_{cc}) = \exp 0.36 = 1.4333$$

Rate of return is 43.33%

Question

- Question: A bank offers two alternative interest schedules for a savings account of $\$100{,}000$ locked in for 3 years: (a) a monthly rate of 1% and (b) an annually, continuously compounded rate, r_{cc} of 12%. Which alternative should you choose?